
# JEE Main Maths

## Short Notes

### Three Dimensional Geometry

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**1. The distance between two points in three-dimension cartesian system**

The distance between two points P $(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in the three-dimensional Cartesian system is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

**2. Section formula**

If C(x, y, z) divides the join of $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio $m_1 : m_2(m_1, m_2 > 0)$ then

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2};\ y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2};\ z = \frac{m_1z_2 + m_2z_1}{m_1 + m_2}$$

and

$$x = \frac{m_1x_2 - m_2x_1}{m_1 - m_2};\ y = \frac{m_1y_2 - m_2y_1}{m_1 - m_2};\ z = \frac{m_1z_2 - m_2z_1}{m_1 - m_2}$$

**3. Centroid of a Triangle**

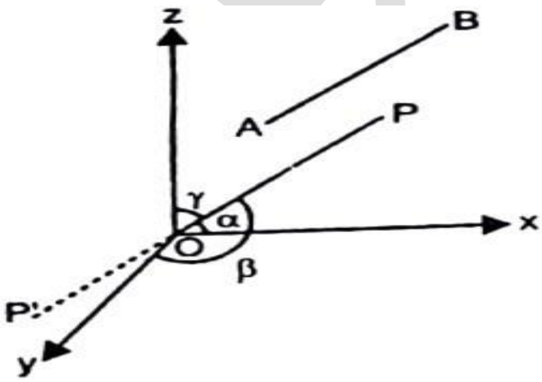
The centroid of a triangle XYZ whose vertices are $X(x_1, y_1, z_1)$, $Y(x_2, y_2, z_2)$ and $Z(x_3, y_3, z_3)$ are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

**4. Centroid of a Tetrahedron**

The centroid of a tetrahedron ABCD whose vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ are $\left(\frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{y_1 + y_2 + y_3 + y_4}{4}, \frac{z_1 + z_2 + z_3 + z_4}{4}\right)$

**5. Direction Cosines (d.c's)**



If a line makes in angles α, β, γ with positive directions of x, y and z-axes then cos α, cos β, cos γ are called the direction cosines of the line. Generally, direction cosines are represented by l, m, n.

The angle α, β, γ are called the direction angles of the line XY and the direction cosines of YX are cos(π – α), cos(π – γ) i.e., – cos α, –cos β, –cos γ.

Thus, the direction cosines of the x-axis are cos 0, cos π/2, cos π/2 i.e., 1, 0, 0. Similarly the d.c's of y and z axis are (0, 1, 0) and (0, 0, 1) respectively.

**Note:**

**(a)** If ℓ, m, n be the d.c's of a line OP and OP = r, then the coordinates of the point P are (ℓr, mr, nr).

**(b)** $\ell^2 + m^2 + n^2 = 1$ or $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

**(c)** $\text{Sin}^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

**6. Direction ratios (d.r.'s)**

Direction ratios of a line are numbers which are proportional of the d.c's of a line.

Direction ratios of a line PQ, (where P and Q are (x1, y1, z1) and $(x_2, y_2, z_2)$ respectively, are $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$.

**7. The relation between the d.c.'s and d.r.'s**

If a, b, c and the d.r.'s and l, m, n are the d.c.'s, then

$$\ell = \pm\frac{a}{\sqrt{(a^2+b^2+c^2)}}, m = \pm\frac{b}{\sqrt{(a^2+b^2+c^2)}}, n = \pm\frac{c}{\sqrt{(a^2+b^2+c^2)}}$$

**Remember:** If a, b, c and d.r.'s of AB then d.c's of a line AB are given by the +ve sign and those of the line BA by –ve sign.

**8. The angle between the two lines**

If $(\ell_1, m_1, n_1)$ and $(\ell_2, m_2, n_2)$ be the direction cosines of any two lines and θ be the angle between them, then, **$\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$**

**(a)** If lines are perpendicular then **$l_1 l_2 + m_1m_2 + n_1n_2 = 0$**

**(b)** If lines are parallel then $\frac{\ell_1}{\ell_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

**(c)** If the d.r.'s of the two lines are $a_1, b_1, c_1$ and $a_2, b_2, c_2$ then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1^2 + b_1^2 + c_1^2)}\sqrt{(a_2^2 + b_2^2 + c_2^2)}} \quad \& \quad \sin\theta = \pm\frac{\sqrt{\Sigma(b_1c_2 - b_2c_1)^2}}{\sqrt{(a_1^2 + b_1^2 + c_1^2)}\sqrt{(a_2^2 + b_2^2 + c_2^2)}}$$

So, that the conditions for perpendicular and parallelism of two lines are repetitively.

$a_1a_2 + b_1b_2 + c_1c_2 = 0$ and $\frac{a_1}{a_1} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

**(d)** If $l_1, m_1, n_1$ & $\ell_2, m_2, n_2$ are the d.c.'s of two lines, the d.r.'s of the line which is perpendicular to both of them are $m_1n_2 - m_2n_1$, $n_1l_1 - n_2l_2$, $l_1m_2 - l_2m_1$.

**9. The equation of a plane in three-dimensional geometry**

**(i) General form**

Every equation of first degree in x, y, z represents a plane. The most general equation of the first degree in x, y, z is $\mathbf{ax + by + cz + d = 0}$ where at least one of a,b,c is non-zero.

**Note:**

**(a)** Equation of yz plane is $x = 0$

**(b)** Equation of zx plane is $y = 0$

**(c)** Equation of xy plane is $z = 0$

**(ii) One-point form**

The equation of the plane through $(x_1, y_1, z_1)$ is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

**(iii) Intercept from**

The equation of the plane in terms of intercepts of a,b,c from the axes is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

**(iv) Normal form**

The equation of plane on which the perpendicular from the origin of length p and the direction cosines of perpendicular are cos α, cosβ and cos γ with the positive directions of x, y & z-axes respectively is given by $\mathbf{x\cos\alpha + y\cos\beta + z\cos\gamma = p}$

**(v) The equation of the plane passing through three given points**

The equation of the plane passing through $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x-x_2 & y-y_2 & z-z_2 \\ x-x_3 & y-y_3 & z-z_3 \end{vmatrix} = 0$$

**(vi) The equation of a plane passing through a point and parallel to two lines**

The equation of the plane passing through a point $P(x_1, y_1, z_1)$ and parallel to two lines whose d.c's and $l_1$, $m_1$, $n_1$ and $l_2$, $m_2$, $n_2$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$$

**(vii) The equation of a plane passing through two points and parallel to all line**

The equation of the plane passes through two point $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ and is parallel to a line whose d.c.'s are l, m, n is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell & m & n \end{vmatrix} = 0$$

**10. Angle Between two Planes**

If θ be the angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ then

$$\theta = \cos^{-1}\left(\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2)}}\right)$$

**(a)**If planes are perpendicular then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

**(b)** If planes are parallel then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

**11. Angle Between a Plane and a Line**

If α be the angle between the normal to the plane and a line then 90° – α is the angle between the plane and the line.

**12. Length of Perpendicular from a Point to a Plane**

The length of perpendicular from $(x_1, y_1, z_1)$ on $ax + by + cz + d = 0$ is

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{(a^2 + b^2 + c^2)}}$$

**13. Positions of Points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ relative to a Plane**

If the points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ are on the same side or opposite side of the plane $ax + by + cz + d = 0$ then

$$\frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d} > 0 \text{ or } \frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d} < 0$$

**14. The distance between the Parallel Planes**

Let two parallel planes be $ax + by + cz + d = 0$ and $ax + by + cz + d_1 = 0$

**Direct Formula:** The distance between parallel planes is

$$\frac{|d - d_1|}{\sqrt{(a^2 + b^2 + c^2)}}$$

**Alternate Method:** Find the coordinates of any point on one of the given planes, preferably putting $x = 0$, $y = 0$ or $y = 0$ $z = 0$ or $z = 0$, $x = 0$. Then the perpendicular distance of this point from the other plane is the required distance between the planes.

**15. Family of Planes**

Any plane passing through the line of inter-section of the planes $ax + by + cz + d = 0$ and $a_1x + b_1y + c_1z + d_1 = 0$ can be represented by the equation.

$(ax = by + cz + d) + \lambda (a_1x + b_1y + c_1z + d_1) = 0$

**16. Equations of Bisectors of the Angles between two Planes**

Equations of the bisectors of the planes

$P_1 : ax + by + cz + d = 0$ & $P_2 : a_1x + b_1y + c_1z + d_1 = 0$

(where $d > 0$ & $d_1 > 0$) are

$$\frac{(ax+by+cz+d)}{\sqrt{(a^2+b^2+c^2)}} = \pm \frac{(a_1x+b_1y+c_1z+d_1)}{\sqrt{(a_1^2+b_1^2+c_1^2)}}$$

| **Conditions** | **Acute angel Bisector** | **Obtuse angle Bisector** |
|---|---|---|
| $aa_1 + bb_1 + cc_1 > 0$ | – | + |
| $aa_1 + bb_1 + cc_1 < 0$ | + | – |

**17. The Image of a Point with respect to Plane Mirror**

The image of $A(x_1, y_1, z_1)$ with respect to the plane mirror $ax + by + cz + d = 0$ be $B(x_2, y_2, z_2)$ is given by

$$\frac{x_2-x_1}{a} = \frac{y_2-y_1}{b} = \frac{z_2-z_1}{c} = \frac{-2(ax_1+by_1+cy_1+d)}{(a^2+b^2+c^2)}$$

**18. The feet of perpendicular from a point on a plane**

The feet of perpendicular from a point $A(x_1, y_1, z_1)$ on the $ax + by + cz + d = 0$ be $B(x_2, y_2, z_2)$ is given by

$$\frac{x_2-x_1}{a} = \frac{y_2-y_1}{b} = \frac{z_2-b_1}{c} = \frac{-(ax_1+by_1+cz_1+d)}{(a^2+b^2+c^2)}$$

**19. Reflection of a plane on another plane**

The reflection of the plane $ax + by + cz + d = 0$ on the place $a_1x + b_1y + c_1z + d_1 = 0$ is

$2(aa_1 + bb_1 + cc_1), (a_1x + b_1y + c_1z + d_1)= (a_1^2 + b_1^2 + c_1^2)(ax+by+cz+d)$

**20. Area of a Triangle**

If $A_{yz}$, $A_{zx}$, $A_{xy}$ be the projections of an area A on the co-ordinate planes yz, zx and xy respectively, then

$$A = \sqrt{(A_{yz}^2 + A_{zx}^2 + A_{xy}^2)}$$

If vertices of a triangle are $(x_1, y_1, z_1)$, $(x_2, y_2, z_2)$ and $(x_3, y_3, z_3)$ then

$$A_{yz} = \frac{1}{2}\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}$$

$$A_{zx} = \frac{1}{2}\begin{vmatrix} z_1 & x_1 & 1 \\ z_2 & x_2 & 1 \\ z_3 & x_3 & 1 \end{vmatrix}$$

$$A_{xy} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

**Note** The area of triangle = ½ bc sin A.

**21. Pair of Planes: Homogeneous Equation of Second degree**

An equation of the form $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ is called a homogeneous equation of second degree. It represents two planes passing through the origin. The condition that it represents a plane is $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

**22. Angle between two Planes**

If θ is the acute angle between two planes whose joint equation is $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$, then

$$\theta = \tan^{-1}\left\{\frac{2\sqrt{f^2 + g^2 + h^2 - bc - (ca - ab)}}{a + b + c}\right\}$$

**Note:** If planes are perpendicular then $a + b + c = 0$

**23. General equation of a straight line**

Let $ax + by + cz + d = 0$ and $a_1x + b_1y + c_1z + d_1 = 0$ be the equations of any two planes, taken together then $\mathbf{ax + by + cz + d = 0 = a_1x + b_1y + c_1z + d_1}$ is the equation of straight line.

The x-axis has equations $y = 0 = z$, the y-axis $z = 0 = x$ and the z-axis $x = 0 = y$.

**24. Equation of a line Passing through a Point and Parallel to a Specified Direction**

The equation of the line passing through $(x_1, y_1, z_1)$ and parallel to a line whose d.r.'s an a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = r$$

and the co-ordinate of any point on the line an $(x_1 + ar, y_1 + br, z_1 + cr)$ when r is directed distance.

**25. The equation of line Passing through two Points**

The equations of the line passing through $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

**26. Symmetric Form of the equation of the line**

The equation of the line passing through $(x_1, y_1, z_1)$ and having direction cosines l, m, n is

$$\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

**27. To convert from General Equation of a line to Symmetrical Form**

**(i) Point:** Put x = 0 (or y = 0 or z = 0) in the given equations and solve for y and z.

The values of x, y and z are the co-ordinates of a point lying on the line.

**(ii) Direction cosines:** Since the line is perpendicular to the normal to the given planes then find direction cosines. Then write the equation of the line with the help of a point & direction cosines.

**28. Angle between a Line and a Plane**

If angle between the line $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = r$ and the plane $a_1x + b_1y + c_1z + d = 0$ is θ then 90° – θ is the angle between normal and the line

i.e., $$\cos(90° - \theta) = \frac{(aa_1 + bb_1 + cc_1)}{\sqrt{(a^2 + b^2 + c^2)}\sqrt{(a_1^2 + b_1^2 + c_1^2)}}$$

or

$$\sin\theta = \frac{(aa_1 + bb_1 + cc_1)}{\sqrt{(a^2+b^2+c^2)}\sqrt{(a_1^2+b_1^2+c_1^2)}}$$

**Note:** If line is parallel to the plane then $aa_1 + bb_1 + cc_1 = 0$

**28. General Equation of the Plane containing the Line**

$$\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$ is $a(x - x_1) + b(y - y_1) + c(z - z_1)$ where $al + bm + cn = 0$

**29. Coplanar lines**

**(i) Equations of both lines in symmetrical form**

If the two lines are $\frac{x-x_1}{\ell_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ & $\frac{x-x_2}{\ell_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$

Coplanar then
$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix} = 0$$

& the equation of plane containing the line is

**(ii) If one line in symmetrical form**

Let lines are $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$

The condition for coplanarity is

$$\frac{a_1x_1 + b_1y_1 + c_1z_1 + d_1}{a_2x_1 + b_2y_1 + c_2z_1 + d_2} = \frac{a_1\ell + b_1m + c_1n}{a_2\ell + b_2m + c_2n}$$

**(iii) If both lines are in General form**

Let lines are $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$

and $a_3x + b_3y + c_3z + d_3 = 0 = a_4x + b_4y + c_4z + d_4$

The condition that this pair of liens is coplanar is

$$\begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} = 0.$$

**30. Shortest Distance**

Two straight lines in space when they are not coplanar are called skew lines. Thus skew lines are neither parallel nor intersect at any point. Let PQ and RS are two skew lines and a line which is perpendicular to both PQ and RS. Then the length of the lines is called the shortest distance between PQ and RS.

Let equations of the given lines are

$$\frac{x-x_1}{\ell_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \;\&\; \frac{x-x_2}{\ell_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$$

Let S.D. lie along the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-y}{n}$$

S.D. = $|l(x_2 - x_1) + m\,(y_2 - y_1) + n(z_2 - z_1)|$

and

**Equation of the shortest distance is**

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ \ell_1 & m_1 & n_1 \\ \ell & m & n \end{vmatrix} = 0$$

**and**

$$\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ \ell_2 & m_2 & n_2 \\ \ell & m & n \end{vmatrix} = 0$$

**31.Volume of Tetrahedron**

If vertices of tetrahedron are $(x_1, y_1, z_1)$, $(x_2, y_2, z_2)$, $(x_3, y_3, z_3)$ and $(x_4, y_4, z_4)$ is

$$\frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$$

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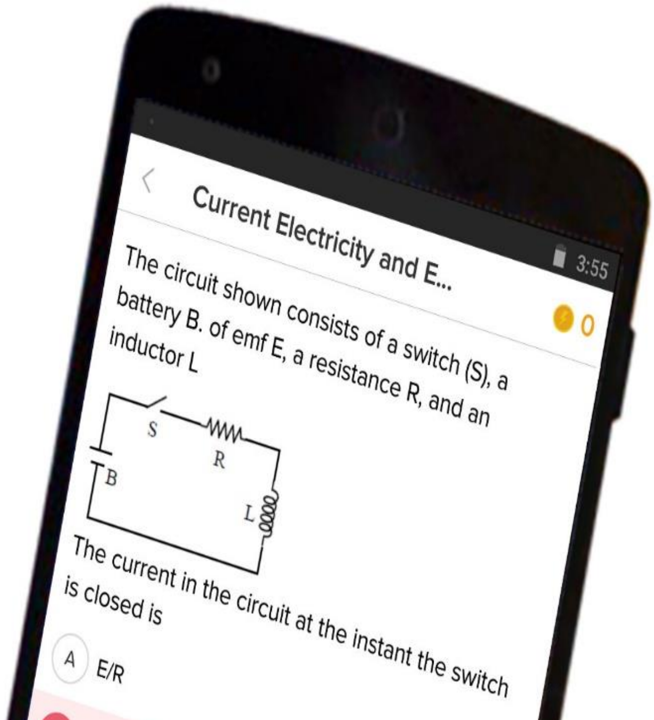

3:55
Current Electricity and E...
0
The circuit shown consists of a switch (S), a battery B. of emf E, a resistance R, and an inductor L
S
R
B
L
The current in the circuit at the instant the switch is closed is
A
E/R